Diffusion Thomson incohérente
Incoherent Thomson scattering

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Based on:
experience in the linear plasma generator Pilot-PSI at Rijnhuizen
Lecture Thomson scattering at TUE by Tony Donné (FOM)
Outline

- Theory with “considerations” from a practical point of view
  - Scatter principles, relation scatter spectrum and VDF / EDF, Rayleigh scattering....
- Examples from fusion and low temperature plasmas
Introduction

- Thomson scattering was described first in 1903 by J.J. Thomson (without a p!!!)
- First application to a laboratory plasma in 1963 by Fünfer (shortly after the discovery of the ruby laser in 1960)
- First measurements in hot plasmas by Peacock et al., in 1969 at the Russian T3 tokamak
The general idea: measure $n_e$ and $T_e$

Light scattering by electrons:
- Scattered intensity reflects $n_e$
- Doppler broadening reflects thermal motion, i.e. $T_e$
Electron moving at $v_e$ ‘sees’ frequency
Observer measures frequency
Shifts add according to scatter vector as

$$\omega_e = \omega_0 - k_0 \cdot v_e$$
$$\omega_s = \omega_e + k_s \cdot v_e$$
$$k = k_s - k_0$$
$$\Delta \omega_s = k \cdot v_e \sim 2k \sin(\theta/2)$$

(for low $v_e$)
Consideration 1:

Doppler shift (i.e. broadening scatter spectrum)

$$\Delta \omega_s \sim 2 k \sin(\theta/2)$$

→ Doppler shift maximal for backward scattering, zero for forward scattering

(velocity sensitivity)
Consideration 1^a:

Doppler shift induced by particle velocity along $k$

→ Thomson scattering probes 1D velocity distribution along $k$, thus determined by selection of $k_0$ and $k_s$

Interesting! Possible arrangement for measuring drift velocity
Incident e.m. wave

\[ E = E_0 \cos(k_0 \cdot r - \omega_0 t) \]

accelerates electron

\[ \ddot{r}_e = \frac{e}{m} E_0 \cos(k_0 \cdot r - \omega_0 t) \]

induces far field radiation

\[ E_s(R, t) = \frac{r_0^2 E_0 \sin \phi}{R} \cos(k \cdot r - \omega_0 t) \]

differential scatter cross section

\( \frac{d\sigma_T}{d\Omega} = r_0^2 \sin^2 \phi \)

(power in direction / \( k_s \) total power)

with classical electron radius

\[ r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2} = 2.82 \times 10^{-15} \text{ [m]} \]
Consideration 2:

\[
\frac{d\sigma_T}{d\Omega} = r_0^2 \sin^2 \phi
\]

Re-radiation is maximum at $\phi = 90^\circ$ (in a plane perpendicular to $E_0$)

$\Rightarrow$ No radiation emitted parallel to the polarization of the incident wave.
Consideration 3:

\[
\frac{d\sigma_T}{d\Omega} = r_0^2 \sin^2 \phi
\]

\[
r_0 = \frac{e^2}{4\pi \varepsilon_0 mc^2} = 2.82 \times 10^{-15} \text{ [m]}
\]

Scattering cross section is very small: \(8 \times 10^{-30} \text{ m}^{-2}\)

⇒ Fight for every scattered photon
Consideration 4:

\[ \frac{d\sigma_T}{d\Omega} = r_0^2 \sin^2 \phi \]

\[ r_0 = \frac{e^2}{4\pi \varepsilon_0 mc^2} = 2.82 \times 10^{-15} [\text{m}] \]

Since \( r_0 \sim 1/m \), the scattering by ions is a factor \((m_e/m_i)^2\) smaller.

\[ \implies \text{Power scattered by ions is negligible} \]
The scatter formula & consideration 5:

\[ P_s = P_0 \frac{d\sigma_T}{d\Omega} n_e \Delta L \Omega S(k, \omega) \]

with

- \( P_0 \) : Incident power
- \( n_e \) : Electron density
- \( \Delta L \) : Length of scattering volume
- \( S(k, \omega) \) : Scattering form factor

\( S(k, \omega) \) describes frequency shifts from electron motion as well as correlation between electrons.

\[ \Rightarrow \] Intensity of scattered light proportional to \( n_e \)
Form factor for Maxwellian velocity distrib.

\[
S(\mathbf{k}, \omega) = \int_{-\infty}^{+\infty} F_k(v_k) \delta[\omega_0 - \omega_s(v)] dv_k
\]

δ–function takes care that each velocity leads to a Doppler-shifted frequency:
\[
\omega_s(v) = \omega_0 + \mathbf{k} \cdot \mathbf{v}
\]

When \( F_k(v_k) \) along \( \mathbf{k} \) is Maxwellian:
\[
F_k(v_k) = \frac{1}{a \sqrt{\pi}} \exp[-(v_k / a)^2]
\]

with thermal velocity:
\[
a = \sqrt{2k_B T_e / m_e}
\]

one finds:
\[
S(\lambda_s) = \frac{1}{\Delta \lambda_e \sqrt{\pi}} \exp[-(\frac{\lambda_s - \lambda_0}{\Delta \lambda_e})^2]
\]
Consideration 6:

\[ S(\lambda_s) = \frac{1}{\Delta\lambda_e \sqrt{\pi}} \exp\left[-\left(\frac{\lambda_s - \lambda_0}{\Delta\lambda_e}\right)^2\right] \]

⇒ Maxwellian velocity distribution gives Gaussian scatter profile of which the width is related to \( T_e \)

\[ \Delta\lambda_e = 2\lambda_0 \frac{a}{c} \sin \frac{\theta}{2} = \frac{2\lambda_0}{c} \sin \frac{\theta}{2} \sqrt{\frac{2k_BT_e}{m_e}} \]
Remark: relativistic effects in fusion plasmas

Relativistic effects become important for $T_e > 0.5$ keV:
- Reduction of cross section due to mass defect by a factor $1/\gamma^2$ (= 0.99 for $T_e = 2.6$ keV)
- Headlight effect: electron radiates preferentially in the forward direction

![Graph showing blue shift due to headlight effect in tokamak spectra](image-url)
What if the VDF is not Maxwellian?

\[ F_v(v) = F_x(v_x)F_y(v_y)F_z(v_z) \]

\[ F_x(v_x) = \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z F_v(v) \]

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Consideration 7:

⇒ Scatter spectrum probes $F_k(v_k)$, so requires deconvolution to retrieve EEDF or EVDF

In fact a two-dimensional Abel inversion, but simpler, namely by differentiation:

$$F_v(v) = -\frac{F_x(v)}{2\pi v}$$
For completeness: from $F_v(v)$ to $f_E(E)$

Integration of the three dimensional velocity distribution over spherical shell gives VDF:

$$f_v(v) = F_v(v) \cdot 4\pi v^2$$

EEDF follows from transformation from v to E:

$$f_E(E) = f_v(v) \cdot \frac{dv}{dE} = f_v\left(\sqrt{\frac{2E}{m_e}}\right) \cdot \frac{4\pi \sqrt{2E}}{m_e^{3/2}}$$

Transformation from 1D velocity distribution:

$$f_E(E) = -\frac{2}{m_e} F_x'(\sqrt{\frac{2E}{m_e}})$$
Example: effect of missing high energy tail

Electrons with velocity $v$ contribute equally to the entire 1D velocity distribution for velocities lower than $v$.

\[ E_0 = \frac{1}{2} m_e v_0^2 \]

\[ f(E) \text{ (eV$^{-1}$)} \]

\[ v_x \text{ (10$^6$ m s$^{-1}$)} \]
Example: effect of missing high energy tail

Difficult to diagnose the high energy tail of the EEDF on basis of noisy Thomson scatter spectra

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Thesis H vd Meiden, TUE 2011
What about *coherent* Thomson scattering?

Waves scattered by electrons that are $2\pi/k$ along $k$ apart are in phase.
Coherent scattering if the motion of these electrons is correlated.
This can per definition only be at distances $>\lambda_D$ (due to ion motion or electron density fluctuations)
Salpeter parameter:

Compares scatter vector and Debye length

Indicates whether coherent behavior of the electrons can be observed

\[
\alpha = \left( k \lambda_D \right)^{-1} = \frac{\lambda_0}{4 \pi \lambda_D \sin \frac{\theta}{2}}
\]

- \( \alpha << 1 \): Scattering by individual electrons
  - Incoherent Thomson scattering
- \( \alpha \geq 1 \): Scattering by (electrons surrounding) ions
  - (Ion) Coherent Thomson scattering
Rayleigh & Raman scattering

- Elastic scattering by bound electrons
- Useful for calibration of Thomson scattering experiments
- Might overwhelm in low temperature weakly ionized plasma
- Much smaller Doppler broadening due to high mass
- Inelastic scattering = Raman scattering

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Rayleigh scattering cross section

- Cross section strongly wavelength dependent:

\[
\frac{d\sigma_R}{d\Omega} = \frac{4\pi^2}{\lambda_i^4} \left( \frac{\mu - 1}{n_\mu} \right)^2 \cdot (1 - \sin^2 \theta \cos^2 \varphi)
\]

with \( n_\mu \) the gas density at which the refractive index \( \mu \) is measured

<table>
<thead>
<tr>
<th>Particle</th>
<th>( \frac{d\sigma_R}{d\Omega} ) (10^{-32} m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>5.40</td>
</tr>
<tr>
<td>Ar(^+)</td>
<td>2.12</td>
</tr>
<tr>
<td>He</td>
<td>0.087</td>
</tr>
<tr>
<td>N(_2)</td>
<td>6.07</td>
</tr>
<tr>
<td>O(_2)</td>
<td>4.99</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>13.6</td>
</tr>
</tbody>
</table>
Summary

- Intensity of scattered light $\sim n_e$
- For Maxwellian VDF, width of scattered spectrum $\sim T_e$
- Scatter vector determines which 1D velocity distribution is probed
- No scattering parallel to polarization incident wave
- Scattering yield is very low: $P_s/P_0 \sim 2 \times 10^{-15}$ for $n_e = 5 \times 10^{19}$ m$^{-3}$, $\Delta L = 5$ mm, $\Omega = 5 \times 10^{-3}$ sr and 20% transmission of optical system
- Rayleigh scattering provides calibration
- Insight into EEDF, difficult to assess high energy tail
What is a favourable wavelength to perform a Thomson scattering experiment at?

(UV - VIS - IR)
Examples

- JET: Lidar
- Textor: imaging system
- Pilot-PSI / Magnum-PSI
LIDAR

Light Detection And Ranging

features 180° scattering

Time-of-flight information of scattering from short pulse (300 ps \( \propto 9 \) cm) gives position information
LIDAR

Only of interest for large devices
JET is only tokamak with LIDAR
also proposed for ITER

JET system features
6-ch filter spectrometers
with fast Micro-channel
plate detectors

C. Gowers et al., RSI 66 (1995) 471
Spectral analysis in filter spectrometer

Filter spectrometer and corresponding channel distribution at DIII-D
Figures taken from T.N. Carlstom et al., RSI 63 (1992) 4901
Textor: imaging (TV) system

Full laser chord is imaged onto detector

Full chord imaged onto detector

Baffles to reduce stray light cone from windows far from vessel

Baffles to protect detector from "Tokamakium"

Both wings of scattered light spectrum imaged onto detector

ICCD for gating

- Allows to reduce background contribution (plasma light) as well as stray light (time-of-flight effect)

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$T_e$ and $n_e$ profiles as a function of time for a plasma with $m=2$ islands with peaked $n_e$ inside the island
Introduction: Pilot-PSI & Magnum-PSI

Pilot-PSI:
- in operation
- forerunner of Magnum-PSI
- on basis of its extreme plasma conditions already unique in its own right for PSI research

Magnum-PSI:
- first plasma on June 18th 2009
- commissioning started, scientific program from 1-2012

C target exposed to $10^{24}$/m$^2$s H-plasma
Experimental: Pilot-PSI

Thomson scattering

Source: Cascaded arc

\[ H_2 \]

\[ \text{cathode} \]

\[ \text{nozzle/anode} \]

\[ \Delta z = 10 - 40 \text{ mm} \]

\[ \text{H plasma: } 10^{19} - 5 \cdot 10^{21} \text{ m}^{-3}, 0.1-5 \text{ eV} \]

\[ \Delta z = 5 - 500 \text{ mm} \]

\[ 0.55 \text{ m} \]

\[ \text{Target} \]

\[ \text{removable mirror} \]

\[ \text{Coils} \]

\[ \text{pumps} \]
Thomson scattering on Pilot-PSI

**Vertical profiles of** $n_e$ **and** $T_e$ ↔

Nd:YAG laser
0.5 J/pulse, 532 nm, 10 Hz

C viewing dump

laser dump

focusing lens

plasma

fiber (48) array

collection and relay optics

spectrometer

Littrow doublet

2D detector

vertical coordinate (30 mm)

Wavelength (10 nm)

Stray light and Rayleigh signal in Pilot-PSI

Low stray light level:
\[ < 5 \cdot 10^{17} \text{ m}^{-3} \text{ density equivalent} \]

Thomson : Rayleigh \( \sim 10^3:1 \)
\( (p = 1 \text{ Pa}, n_e=10^{20} \text{ m}^{-3}, T_{\text{neutrals}} \sim 1000 \text{ K}) \)

Target exposed to $10^{24} / \text{m}^2\text{s} \ H$-plasma
Source performance: H plasma

Density profiles at 6 slm and 400 A

Temperature profiles at 6 slm and 400 A

Electron densities up to $5 \cdot 10^{21} \text{ m}^{-3}$ at 3 eV

30 lasershots averaged

G. J. van Rooij et al Appl. Phys. Lett.2007 90 121501
Multiple channel sources

- Successful operation of three-channel cascaded arc on argon and hydrogen
- Experiments on mixing & merging of hydrogen beams with ring electrode ongoing

Vijvers et al, Fusion Engineering and Design 25th SOFT special issue
Gerard van Rooij, Orleans, November 23rd 2011
C: Return current

- TS laser
- Return current filament
- Floating body
- Anode
- Central plasma channel
- Plasma light
- Single shot TS
- Return current filament
- Electron density ($10^{20} \text{ m}^{-3}$)
- Vertical position (mm)
TS as routine technique at Pilot-PSI

ITER relevant

Electron density (m⁻³)

Electron temperature (eV)

$10^{24}$ m⁻² s⁻¹

$10^{25}$ m⁻² s⁻¹

$10^{26}$ m⁻² s⁻¹

Target floating

Target grounded

Magnetic field:

- 0.4 T
- 0.8 T
- 1.2 T
- 1.6 T

Operating current:

- > 175 A
- 125 – 175 A
- < 125 A
TS laser beam line on Magnum-PSI

- 2 beam lines
- fast mounting
- 50 mm translation
Stray light reduction in triple grating spectrograph

- Especially relevant when stray light preventing measures are not possible (e.g. measure insight light bulb)

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Stray light suppression with absorption cell

- iodine-vapor filter cell (90-mm-diameter, 230-mm-long) to filter stray light: filter width of the order of the line width of single mode 532 Nd:YAG

Hoffman et al., Optics Letters 21, 1996